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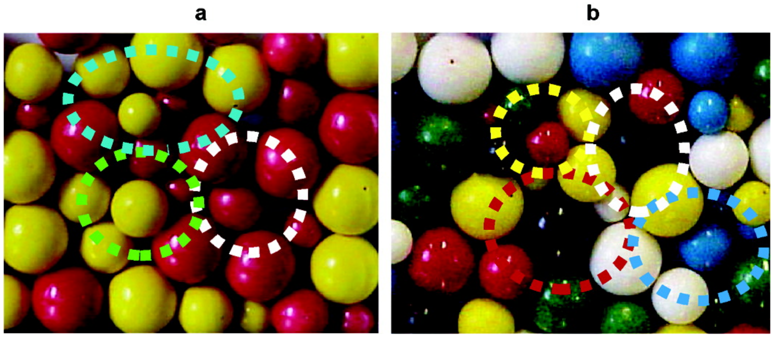

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"Single Sample Concept": Theoretical Model for a Combinatorial Approach to Solid-State Inorganic Materials

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A theoretical model for a "single sample concept" (SSC) applied to the combinatorial chemistry of solid-state inorganic compounds is presented. The SSC is performed by reacting N starting materials (randomly mixed) in a single sample of $\sim$1 cm^3. Combinatorial calculations demonstrate that the number of reasonably estimated phases to be found in the space of N components grouped into compounds (e.g. oxides, sulfides, or halogenides) containing up to $q \leq 6$ metallic elements is smaller than combinations set up by starting conditions for local reactions within the bulk of a single ceramic sample. Recently, the SSC proved to work in the case of synthesizing libraries of 3d metal oxides, from which magnetic particulate matter was extracted by a magnetic separation technique. The SSC may be applied in a first screening cycle followed by 2D approaches of combinatorial chemistry.

Introduction

Following up just combinatorial arguments to combine elements for reactions into solid-state compounds, properties of combinatorial functions make it impossible to set up all compositions in real experiments.[1−4] In view of a limited amount of chemicals available, we may conclude to have, therefore, no access to the manifold possibly existing compounds generated by combinatorial functions. However, present knowledge bears no evidence for a realization of most combinatorial compositions by thermodynamically stable compounds:[5] The number of compounds appearing in binary, ternary, etc., phase diagrams is by far not as high as combinatorial variations of elements and stoichiometry would allow.

There are further issues related to the dimension of the phase space we have to enface by synthesis: (i) As reflected by minerals and entries for synthetic compounds in databases, there is no evidence for particularly numerous compounds made of *many* constitutional (q) (for a list of symbols, see Appendix) elements.[2,6] (ii) With respect to physical properties, all known effects (magnetism, ferroelectricity, superconductivity, and many others) were obtained for a *low* number q of constitutional metallic elements ($q \leq 4$; e.g., for metals in oxide materials).[5,6] This let us anticipate that for finding new materials featuring interesting physical properties, we most likely will not need compounds made of, say, q larger than 6 metallic elements.

By reasonable arguments, we may, thus, set an upper limit for the number q of constitutional elements in compounds to be found by combinatorial chemistry. Redefined as such, combinatorial searches for new inorganic solid-state materials become a feasible endeavor, provided combinatorial functions representing the experimental method reach the same order of combinations as those describing a realistic number of existing compounds. In the following chapters, we introduce a theoretical model demonstrating that local reactions taking place within a single ceramic sample (made by mixing up N components) can provide access to existing compounds in a high dimensional phase space.

Present combinatorial techniques rely on 2D plate techniques,[6,7] in which the number of experiments performed in parallel is limited by the number of spots on a plate.

Recently, a single sample concept (SSC) operating on the basis of using a random mixture of N starting materials making up a single sample of $\sim$1 cm^3 was presented.[5] Experimental implementation has demonstrated the syntheses of libraries for ferri- or ferromagnetic oxides. Mixtures made of 17 (Li, Na, Mg, Si, K, Ca, Sr, Y, Nb, Mo, Sn, Te, Ba, La, W, Pb, Bi), 24 (Li, B, Na, Mg, Al, Si, K, Ca, Ti, Ga, Ge, Sr, Y, Zr, Nb, Mo, In, Sn, Te, Ba, La, W, Pb, Bi), and 30 (Li, B, Na, Mg, Al, Si, K, Ca, Ti, Zn, Ga, Ge, Rb, Sr, Y, Zr, Nb, Mo, Cd, In, Sn, Te, Cs, Ba, La, Ta, Hf, W, Pb, Bi) nonmagnetic oxides containing 5−25 wt % of Fe_2O_3 (all others had the same amount in wt %) were pressed into pellets and reacted at $T = 850$ °C (for 4 h, $p(O_2) = 1$ atm), to effect local reactions between all kinds of combinations of grains represented by $N + 1$ different elements. After the reactions, ball milling was used to liberate the manifold possible compounds to prepare an emulsion to pass through a magnetic separation column. Through the effect of a magnetic field gradient, ferri- or ferromagnetic particles were extracted up to 75 wt % for 25 wt % of Fe_2O_3, and $N = 17$. Single-grain analyses performed by scanning electron microscopy revealed a library consisting of individual grains representing magnetic Fe oxides. Bulk susceptibility measurements confirmed hystersis loops. To improve the formation of single grains, reactions were also performed using NaCl (90 wt %) particles loaded by oxides (10 wt %). In the case of cuprates, single sample reactions as described

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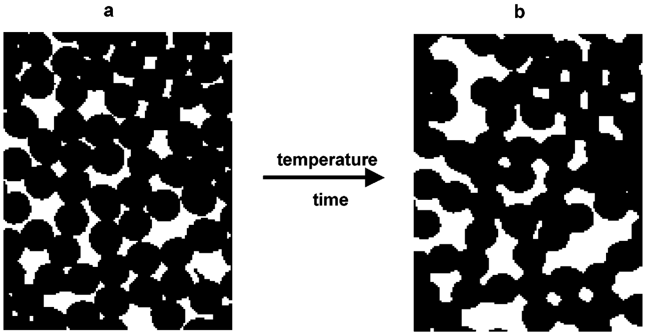

Figure 1. Displacement and states of densification of spherical copper particles of the same size. (a) 1 min at $T = 1273$ K, (b) 12 h at $T = 1276$ K. Original diameter, 30 μm. (a) Binary contacts present at the beginning, on time more and more transform into a clustered state (b). After ref 9 a.

above (N: Ca, Sr, Y, Ba, La, Tl, Pb, Bi, Cu) have produced superconductivity up to 100 K (confirmed by measuring the resistance and a Meissner effect).[8] For more details on the preparation and analysis of magnetic libraries, see ref 5.

The SSC: Assumptions and Basic Ideas

The present concept uses ceramic samples, that is, an assembly of densely packed grains of many (N) starting materials, which are brought to reaction by thermal activation (calcination). As known from studies on the densification of assemblies of grains[9] (Figure 1), we can assume local reactions between grains in spatial proximity.

In traditional ceramic synthesis, just a small number N of components ($N \sim 2-4$) are brought to reaction.[9] Consequently, local densification can produce a stable product at nearly any place within a sample, at least after a long time of reaction. By SSC, a large number N of starting materials are randomly mixed in order to effect local reactions which may produce a high variety of products across just one sample of a volume of $\sim$1 cm^3. This is because of a randomly constituted neighborhood of grains (called a local configuration: C) representing different elements and corresponding masses. In Figure 2, we show randomly mixed spheres in 2D to illustrate local configurations in particulate matter (panel a: $N = 2$; panel b: $N = 6$).

A local configuration is thus made up of n grains of q different elements which can locally react to a compound $\mathbb{R}(q, X_i, \mathrm{L}) \equiv E_{X_1}(1)E_{X_2}(2)E_{X_3}(3)\cdots E_{X_q}(q)E_{X_\mathrm{L}}(\mathrm{L})$ (E_L: lead element, e.g., oxygen, sulfur, or halogens).

Local reactions

$$C(q, n) \xrightarrow{T, p, t} \mathbb{R}(q, X_i, \mathrm{L}) \text{ (where } i = 1, 2, ..., q;\ n \geq q)$$

produce a library, obtained at conditions of T; p; and reaction time, t.

Breaking crystallites apart by milling reacted ceramic samples liberates single-phase and intergrown crystallites, which can be characterized by local probe techniques (electron microscopy), providing a lateral resolution of a few hundred nanometers.

The SSC is particularly of interest for searching materials featuring magnetic properties (ferri- or ferromagnetic or superconducting) because of the application of magnetic separation techniques[10-12] to extract minor quantities of grains retained by magnetic separation columns.[5]

Conceptually, SSC may be performed in two ways: (i) N starting materials are mixed in equal portions with no preference for a metallic lead element (E_L). (ii) Adding a larger quantity of a metallic E_L statistically enhances formation of products containing element E_L. Presently, we apply path (ii) for searching ferri-/ferromagnetic or superconducting metal oxides of E_L (Fe, Co, ..., etc; Cu, respectively) formed by compounds involving other nonmagnetic elements. For a brief summary on first results, see the Introduction.

Summarizing, SSC is performed by following up a scheme as shown in Figure 3. In the search for new magnetic or superconducting materials, SSC[5] may thus be applied as a primary screening, followed by a 2D secondary screening mode. However, as indicated by arrows, SSC bears the possibility of a convergent procedure, reducing the number of starting materials (N) per cycle.

Combinatorial Model

Given N chemically different components (starting materials), the number N_q^{PD} of phase diagrams constituted by q components ($q \in N$) is equal to the number of combinations made up by q elements selected out of N (multiple selections of a single element being not allowed).[13]

$$N_q^{\mathrm{PD}} = \frac{N!}{q!(N-q)!} \tag{1}$$

Each of these phase diagrams contains a number η_q of phases involving q elements. On average, phase diagrams of the order q contain $\langle \eta_q \rangle$ phases, representing basic structural types of compound classes to be found. The total number of phases N^P ($q \geq 2$) in SSC samples attempted to produce is given by

$$N^\mathrm{P} = \sum_{i=2}^{q} \langle \eta_i \rangle N_i^{\mathrm{PD}} \tag{2}$$

The number of experiments set up by whatever combinatorial procedure has thus to provide combinations for starting materials that are at least as numerous as N^P, including a broad variation in the number of locally available components (stoichiometry).

The basic requirement is met by the number N_n^C of local configurations C, which reads as

$$N_n^\mathrm{C} = \binom{N+n-1}{n} - Nn \tag{3}$$

(combinations including multiple selections of the same element).

Here, n is the number of grains associated with a particular $C(q, n)$ ($n \geq q$).The second term in eq 3 accounts for those configurations that contain a single element; these do not yield products.

Variable n is a characteristic quantity representing the local degree of close packing of grains in an unreacted ceramic sample. The occurrence of $C(q, n)$s follows a distribution for which we define an average number $\langle n \rangle$ of grains in $C(q, n)$s.

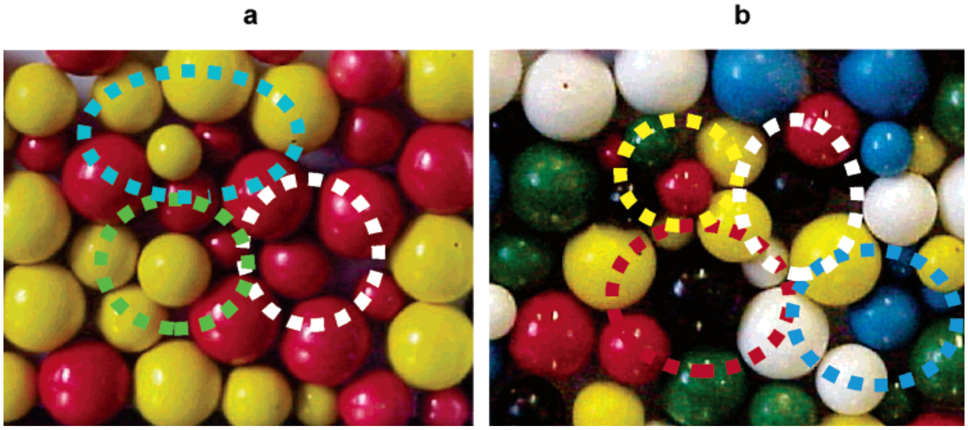

Figure 2. 2D random packing of spheres of different colors and sizes: (a) two components, (b) six components. Indicated are local arrangements of spheres in close proximity (called a local configuration, *C*). It is assumed that locally products are formed upon configurations of particles. Note that particles may belong to different configurations (sharing mass through reactions).

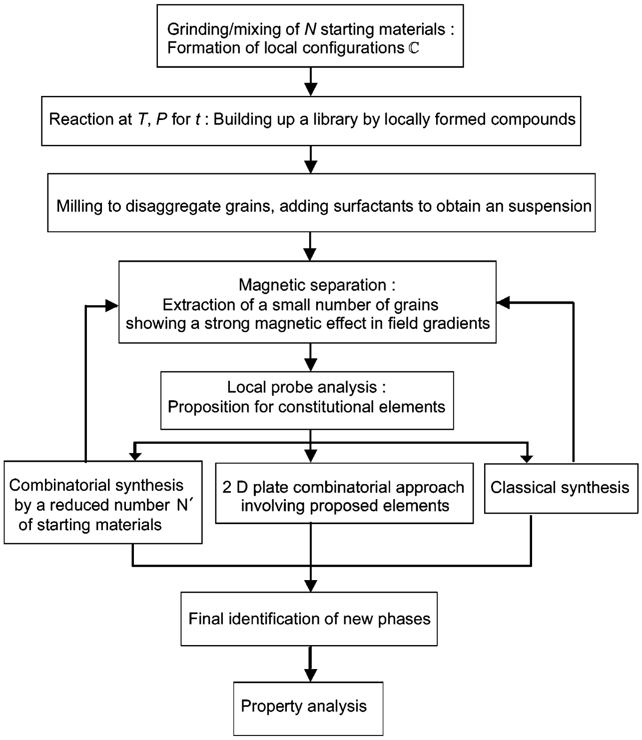

Figure 3. Schematic summary of how to process the single sample concept in the case of a property-oriented search for magnetic materials.

An estimation of $\langle n\rangle$ for a real sample may be obtained by models developed in the case of randomly packed spheres of equal size.[14−16] Figure 4 was generated from literature data[14] showing a correlation between the mean voidage $\langle \epsilon\rangle$ (porosity) and the average coordination number $\langle \tilde{n}\rangle$ (number of spheres around a selected one being in close contact with neighbors). For various types of random packs of equal spheres, $\langle \epsilon\rangle$ varies in the range of 0.36−0.44.[14] However, grains in SSC samples are pressed into dense pellets. In this case, we can assume a smaller $\langle \epsilon\rangle$ than found in studies of the packing of spheres. Consequently, we assume a $\langle n\rangle = \langle \tilde{n}\rangle + 1$ above a value of 8 (see Figure 4).

For providing a large reservoir of grains from which configurations randomly form during grinding/mixing of components, a grain size in the range of 10 down to 1 μm is suitable, because in 1 cm^3, there are $\sim 10^9 - 10^{12}$ grains,[17] respectively. A high number of starting grains allows one to set up possible configurations many times. The realization of configurations by a distribution for the grain size provides stoichiometric variation. This is considered very important, because stochiometic variation is needed to explore corresponding phase diagrams defined by *q* elements within a configuration. Exploration of phase diagrams of the order *q* becomes effective also through $C(q, n)$s ($n > q$), even for $\langle \eta_q\rangle$ values that are fairly high.

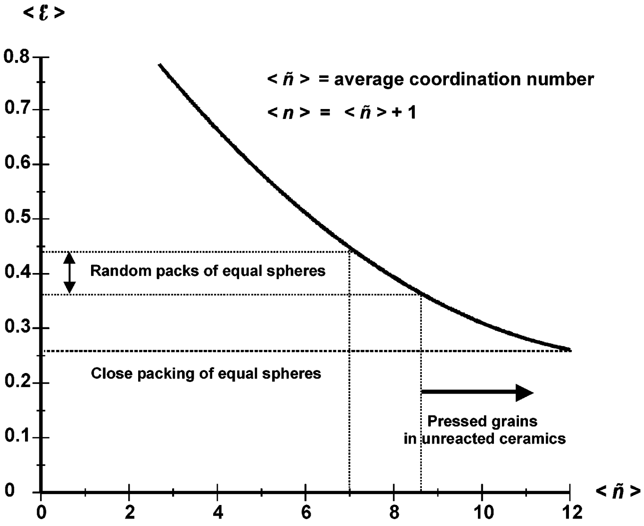

Figure 4. Correlation between the voidage (porosity) $\langle \epsilon\rangle$ and the coordination number $\langle \tilde{n}\rangle$ for randomly packed spheres of equal size.[17] For pressed unreacted ceramic samples, we can assume a $\langle \tilde{n}\rangle$ larger than found for studies on spheres on which no pressure was applied.

Because the SSC does not attempt to produce large quantities of single-phase products, minor quantities of new phases can be obtained by population of a limited number of trajectories in phase diagrams of the order *q*. This is because phases $\mathbb{R}_i$ ($i = 1, 2, ..., \eta_q$) appear in compositional domains (fields, volumes, and higher dimensional space) that are more likely to hit by a random choice of compositions than by finding lines or points. In Figure 5, we illustrate how randomly selected compositions (through $C(q, n)$s with $n > q$ or the grain size distribution) may fall into existence fields of phases: For producing a minor quantity of phases $\mathbb{R}_1$, $\mathbb{R}_2$, and $\mathbb{R}_3$, in principle, just a small number of trials is

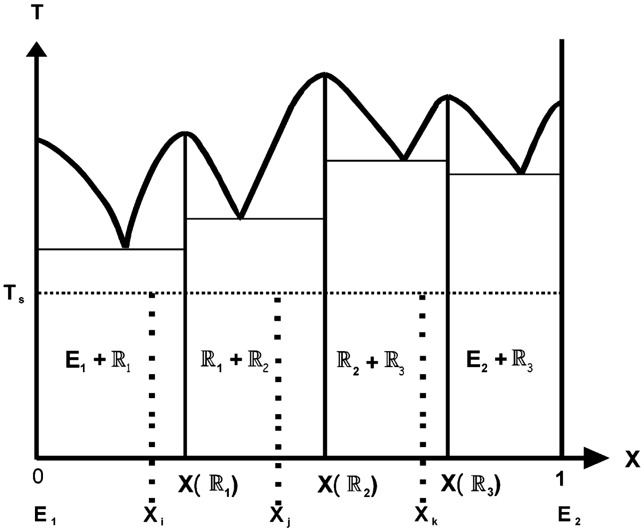

Figure 5. A schematic binary phase diagram of components E_1 and E_2 showing the existence fields of three phases ($\mathbb{R}_1$, $\mathbb{R}_2$, and $\mathbb{R}_3$). Because of existence fields, two randomly selected compositions (X_i and X_k or X_j and X_k) can produce minor quantities of $\mathbb{R}_1$, $\mathbb{R}_2$, and $\mathbb{R}_3$. This illustrates that for finding phases, we do not necessarily need to hit the stoichiometric composition ($X(\mathbb{R}_1)$, $X(\mathbb{R}_2)$, $X(\mathbb{R}_3)$) on the X axis. T_S: subsolidus temperature for reaction.

Table 1. Approximate Numerical Values for Combinatorial Functions Describing the Number of Phase Diagrams and Local Configurations (C) Depending on the Total Number, N, of Components (Metallic Elements), That Is, Oxides, Sulfides or Halogens

N	q	N_q^{PD}	$\langle n\rangle$	N_n^{C}
	4	2.1×10^2	6	4.9×10^3
10			8	2.4×10^4
	6	2.1×10^2	10	9.2×10^4
	4	4.8×10^3	6	1.8×10^5
20			8	2.2×10^6
	6	3.9×10^4	10	2.0×10^7
	4	2.7×10^4	6	1.6×10^6
30			8	3.9×10^7
	6	5.9×10^5	10	6.4×10^8
	4	9.1×10^4	6	8.1×10^6
40			8	3.1×10^8
	6	3.8×10^6 calcd by eq 1	10	8.2×10^9 calcd by eq 3

necessary. Applied to higher order ($q > 2$) diagrams, we can say that the number η_q of existing phases (given T and p) can be explored by a number of compositional combinations that is in the same order as η_q ($q = 2, 3, ...$), however, preferably much larger in real experiments.

In case the number of grains N_E available for each element E is by orders of magnitude larger than necessary to populate each $C(q, n)$, stoichiometric variation is brought into the configurations by distribution for the grain size. A large excess of grains is similarly in favor of a processable number of product grains, that is, grains to be isolated after passing a suspension through a separation column.

Table 1 provides calculated values for N_q^{PD} and N_n^{C}, ($N \leq 40$). Using an average grain size $\langle d\rangle$ in the range of 1 μm, $C(q, n)$s may be realized $\sim 10^8$ ($N = 10$, $n \leq 8$) to 10^4 ($N = 40$, $n \leq 8$) times, depending on the number, N, of starting materials involved.

Going back to arguments developed in the Introduction, it is reasonable to set limits to q and $\langle\eta_q\rangle$, if q is limited to $q \leq 6$ and $\langle\eta_q\rangle$ is smaller than, say, 10−100 (oxides: mean values for $q = 2$, 3 were found in the range of 2−3, respectively).[5] Limiting $\eta_{\max}$ to relatively high numbers as given above take into account a certain number of kinetically controlled phases, which may also form in SSC samples.

We are about to conclude that because in unreacted ceramic samples $\langle n\rangle$ may be above 8, consequently, the number of configurations is larger than the number of phase diagrams (N_q^{PD}), and N_n^{C} is of similar order or even larger than the number, N^{P}, of possibly existing phases.

The SSC is as simple as this: by taking advantage of a large number of local configurations described by a combinatorial function, which for a given number N of elements can surpass the theoretical number of phase diagrams and phases therein. The SSC model is, thus, able to account for (i) the number of phase diagrams set up by a random sample of N components, (ii) the stoichiometric variation to scan these phase diagrams, and (iii) an available number of starting grains in 1 cm^3 to exceed realistic estimations for the number of phases present in the phase diagrams.

Conclusions

The present analysis shows that the SSC can take advantage of combinatorial functions that realistically can exceed those similar functions that describe possibly existing solid-state inorganic compounds. In the case that the chemical systems in the solid state do not show a preference to form thermodynamically stable compounds for high q values, the number of combinatorially possible compounds does not reach those astronomical numbers, which are reported in general discussions on the principal limitations of experimental combinatorial chemistry. Consequently, single sample combinatorial chemistry seems to merge into an endeavor in which we should be able to explore of what is “there”.

In this case, we are right in claiming that important material properties may be obtained by a limited number, q, of constitutional elements in solid-state compounds; there is no real need for attempting compounds exceeding $q \approx 6$ significantly. For property-directed syntheses, application of magnetic separation techniques proved recently[5] to be very efficient in extracting magnetic phases from SSC samples. This is presently being applied to search for ferri- or ferromagnetic copper-based oxides showing a $T_c \geq 300$ K. Knowledge on such phases is of particular interest for the interpretation of experimental data showing similarity to superconductivity.[18]

In summary, SSC may be used in a first screening step followed by a refined cycle using 2D procedures. Present calculations show that within a single ceramic sample, the number of reactions performed in parallel can be by orders of magnitude higher than achieved by present 2D approaches.

Nomenclature

N: number of components (e.g. M_XO_Y) to bring to reaction
N_E: number of starting grains per element
N_q^{PD}: number of phase diagrams (PD) constituted by selecting q elements out of N
N^{P}: total number of phases to be estimated here

q: number of constitutional metallic/semimetallic elements in, for example, oxide compounds to be formed

η_q, $\langle \eta_q \rangle$: number (average) of phases per diagram of the order q

C, $C(q,n)$: local configurations, i.e., assembly of n grains in a powder sample which undergoes a reaction; a configuration is characterized by q and n

N_n^{C}: number of local configurations (C) set up by N different elements

$\langle n \rangle$: average number of grains in a configuration

$\langle \tilde{n} \rangle$: average coordination number, that is, number of grains in Cs around a central particle

$\langle \epsilon \rangle$: mean free space in a random pack of spheres

$\mathbb{R}$, $\mathbb{R}_i$: compounds formed

$\mathbb{R}(q, X_i, L)$: compound $\mathbb{R}$, indicating elements (q), stoichiometry (X_i), and lead element E_{L}

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